UCRL- 100242
PREPRINT

MAXIMIZATION OF DIRECTED ELECTROMAGNETIC RADIATION WITH AN OPTIMIZED ANTENNA

R. M. BEVENSEE

THIS PAPER WAS PREPARED FOR SUBMITTAL TO
1989 INTERNATIONAL UNION OF RADIO SCIENCE INTERNATIONAL
SYMPOSIUM ON ELECTROMAGNETIC THEORY
STOCKHOLM, SWEDEN
AUGUST 14-17, 1989

JUNE 1989

Lawrence Livermore National Laboratory

DISCLAIMER

MAXIMIZATION OF DIRECTED ELECTROMAGNETIC RADIATION WITH AN OPTIMIZED ANTENNA*

R. M. Bevensee

Lawrence Livermore National Laboratory, P.O. Box 5504, Livermore, California 94550, USA

*Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under contract number W-7405-ENG-48.

Abstract

We summarize the results of a study to obtain reasonable upperbounds to the maximum energy density $W_T(\Omega_o)$ per steradian radiated during a time interval (-T,T) in a direction Ω_o and also the maximum instantaneous far-field $|r\mathcal{E}(\Omega_o)|$. The constraints are (1) the antenna elements are driven in series by a circuit of zero stored energy around a simple series resonance, (2) the antenna is enclosed within a spherical working volume of radius $a/\lambda_{min} = 1$, (3) signal frequencies are restricted to $\omega \le \omega_{max} = 2\pi c/\lambda_{min}$, (4) the total energy radiated is 1 J. The answer for $W_T(\Omega_o)$ versus $\omega_{max}T$ is depicted on Figure 3. The maximum $|r\mathcal{E}(\Omega_o, t - r/c)|$ is $5.65\sqrt{\omega_{max}}$ at $t = r/c$ for a two-sided fractional bandwidth B = 0.54 and associated gain $G_o(\Omega_o) \approx 16$.

1. Directed radiation in terms of characteristic terminal modes.

We represent the behavior of an M-port antenna with N radiating dipole modes by a method-of-moments matrix equation $\underset{\sim}{V} = \underset{\sim}{Z}\underset{\sim}{I}$ in the ω-domain. We expand $\underset{\sim}{I}$ in characteristic terminal modes [1], with amplitudes proportional to a driving voltage at one port. Then the far-field can be expanded with the M terminal mode vector functions in the ω-domain [2], and, by Fourier transformation, into the t-domain. Assuming that all these functions are parallel in the far-field, in the optimization direction Ω_o, and that the antenna is lossless (later we will relax this condition), we can maximize $W_T(\Omega_o)$ in the ω-domain subject to total radiated energy W = 1 J and obtain an integral equation for the driving function $\psi(\omega)$. Specializing the response to a two-sided fractional bandwidth B about the center frequencies $\pm\omega_o$ and *assuming* $G_o(\Omega_o) = G_o$ is constant over B, we obtain the integral equation

$$\int_{-B/2}^{B/2} d\mu \, \frac{\sin(\mu-\mu^1)\omega_o T}{\mu-\mu^1}\psi(\mu) = \beta\pi\frac{4\pi}{G_o}\psi(\mu^1), \quad -B/2 \le \mu^1 \le B/2 \qquad (1)$$

$\mu = (\omega \mp \omega_o)/\omega_o$ in the $\pm$ frequency band. $\beta = W_T(\Omega_o)/W$. This is satisfied by the angular prolate spheroidal eigenfunctions $S_{on}(c, 2\mu/B)$, $c = \frac{1}{2}B\omega_o T$. β is maximized by the eigenfunction S_{01} of largest eigenvalue $\lambda_1(c) < 1$. Figure 1 shows a plot of $\lambda_1(c)$ versus c [3]. The largest value of β for given gain G_o is, from (1),

$$\beta_1 = \frac{G_o}{4\pi}\lambda_1(c = \tfrac{1}{2}B\omega_o T) \qquad (2)$$

2. Directed radiation in terms of special eigenfunctions.

We can also express the far-field ω-domain $E(\Omega_o)$ as a matrix product $\underset{\sim}{T}^\dagger\underset{\sim}{M}$ ($\dagger$, Hermitian conjugate), where $\underset{\sim}{M}$ is an N x 1 column matrix of dipole moments and $\underset{\sim}{T}$ is a column matrix of positional phase shifts along the direction Ω_o. The time-average radiated power, P_{rad}, at centerfrequency ω_o can be expressed proportional to $\underset{\sim}{M}^\dagger\underset{\sim}{P}\underset{\sim}{M}$, $\underset{\sim}{P}$ related directly to Real $\underset{\sim}{Z}$. Then gain G_o is

$$G_o = 4\pi\,|\underset{\sim}{T}^\dagger\underset{\sim}{M}|^2 / \underset{\sim}{M}^\dagger\underset{\sim}{P}\underset{\sim}{M} \qquad (3)$$

B is P_{rad}/W_R, where "reactive stored energy" W_R [4] measures the frequency derivative of input reactance X (at a simple-pole series resonance) of a circuit driving all the dipoles in series. B is upperbounded by neglecting any stored energy in the drive circuit. W_R can be expressed for an antenna in a quadratic form $\propto \underset{\sim}{M}^\dagger\underset{\sim}{W}\underset{\sim}{M}$ and B written as

$$B = \underset{\sim}{M}^\dagger\underset{\sim}{P}\underset{\sim}{M} / \underset{\sim}{M}^\dagger\underset{\sim}{W}\underset{\sim}{M} \qquad (4)$$

We have expanded $\underset{\sim}{M}$ in column eigenfunctions $\underset{\sim}{X}_i$ of (real) eigenvalues b_i of the matrix equation

$$\underset{\sim}{P}\underset{\sim}{X} = b\underset{\sim}{W}\underset{\sim}{X} \qquad (5)$$

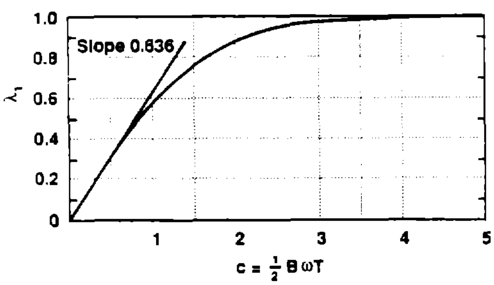

Fig. 1 Largest eigenvalue $\lambda_1(c)$ vs. $c = B\omega T/2$ of Eq. (1).

The result of maximizing B of (4) for a given G_o yields

$$G_o = 4\pi\left[\sum |\underset{\sim}{T}^\dagger\underset{\sim}{X}_i|^2 / (1-\alpha b_i)\right]^2 / \left[\sum |\underset{\sim}{T}^\dagger\underset{\sim}{X}_i|^2\, b_i / (1-\alpha b_i)^2\right] \qquad (6)$$

$$B = \left[\sum |\underset{\sim}{T}^\dagger\underset{\sim}{X}_i|^2\, b_i / (1-\alpha b_i)^2\right] / \left[\sum |\underset{\sim}{T}^\dagger\underset{\sim}{X}_i|^2 / (1-\alpha b_i)^2\right] \qquad (7)$$

α is a running parameter which traces the B-G_o curve from B_{min}-$(G_o)_{max}$, for $\alpha = \pm\infty$ to $(BG_o)_{max}$ for $\alpha = 0$, for B_{max}-$(G_o)_{min}$ for $\alpha = \beta_N^{-1}$ (β_N is the largest eigenvalue).

Equations (6) and (7) should be compared to the corresponding ones for directivity D (= G_o for a lossless antenna) and Q = 1/B in terms of spherical wave modes [5].

If we represent $W_T(\Omega_o)$ and W with the χ_i we obtain (1) again with a redefined $\psi(\mu)$. But now G_o and B in (2) are constrained by (6) and (7) for a given antenna.

3. The best B-G_o curve, spherical working volume of $a/\lambda_{min}=1$

Figure 1 indicates that, with no constraint on centerfrequency ω_o, one should maximize β_1 of (2) with as high a ω_o as possible; i.e., a working volume of largest electrical size. Therefore, we limit ω_o to ω_{max} and choose a so $a/\lambda_{min} = 1$. We chose this value because we have searched for that spatial distribution of short, rather fat dipoles within a sphere of $a/\lambda = 1$ which appears to have the best B-G_o curve (i.e., the highest B for given G_o and vice versa) [6]. That curve is shown on Figure 2, with the expected practical $(G_o)_{max}$ of 50 ≈ ka(ka + 2), supergain limit.

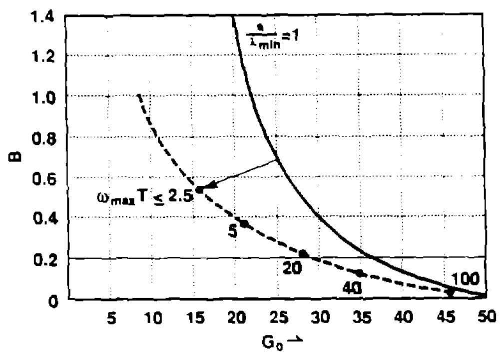

Fig. 2. The best B-G_o curve for $a/\lambda_{min} = 1$ and the scaled B(ω) - $G_o(\omega)$ curve (dashed) for $\omega = \omega_{max} / (1+B/2)$.

That "best" array had the curious distribution of three planes of lines of 0.2 λ dipoles (λ = 1 for convenience), each of radius/length ≈ 0.25 (fat) and overlapping its neighbors by 0.01. With x the direction of Ω_o, one xz plane at y = 0 contained six lines of dipoles in the z direction, extending to the edges of the working sphere, at x = -0.12, 0, 0.12, 0.24, 0.36, and 0.48. The other two xz planes at y = ± 0.43 contained two lines each, at x = 0, 0.12!

Incidentally, this study indicated that P_{rad} given by Re$\mathcal{Z}$ was more accurate than the expression evaluated on the radiation sphere, which developed intolerable error as G_o approached the supergain limit.

4. Maximum $\beta_1(\omega \leq \omega_{max})$ versus $\omega_{max}T$.

It is easy to deduce that one should operate in a simple-pole band as near ω_{max} as possible, rather than in any lower frequency bands. Unfortunately, the a/λ_{min} curve on Figure 2 only describes operation around ω_{max} as centerfrequency. To infer the a/λ-curves for $\lambda > \lambda_{min}$ we have employed the heuristic scaling factors: $|rE_\omega(\Omega_o)|$ ∝ number of dipoles in the volume, ∝ $(a/\lambda)^3$; $G_o \propto (a/\lambda)^2$ because $(G_o)_{max} \lesssim ka(ka+2)$; $P_{rad} \propto |rE_\omega(\Omega_o)|^2/G_o \propto (a/\lambda)^4$; W_R, with a strong $|\bar{r}_1-\bar{r}_2|^{-1}$ dependence between any two dipoles at $\bar{r}_1,\bar{r}_2 \propto (a/\lambda)^3$; and, therefore, $B \propto (a/\lambda)^1$ by (4). With these rules we have plotted on Figure 2 the dashed curve of B(ω) around centerfrequency ω versus $G_o(\omega)$, such that $\omega(1 + B/2) = \omega_{max}$, as follows: a chosen value of B(ω) yields $\omega/\omega_{max} = a/\lambda$, whence $B(\omega_{max}) = B/(a/\lambda)$ on the $a/\lambda_{min} = 1$ curve. From its $G_o(\omega_{max})$, $G_o(\omega) = G_o(\omega_{max}) \cdot (a/\lambda)^2$. Then the point B($\omega$) - $G_o(\omega)$ is known on the dashed curve.

For a given value of $\omega_{max}T$, we maximize β_1 of (2) by searching along the dashed curve in Figure 2 for the B-G_o pair which maximizes $G_o(\omega)\lambda_1(c=\frac{1}{2}B\omega T)$, where $\omega T = \frac{a}{\lambda}\omega_{max}T$. Several points are shown labelled by their $\omega_{max}T$-values, and the values of max$\beta_1(\omega_{max}T)$ are connected by a smooth curve on Figure 3. It is perhaps surprising *how slowly the final value of $\beta_1(\infty) = 50/4\pi \approx 4.0$ is approached.*

To see if ohmic losses would improve the situation we apply the simple theory: loss increases the input power and energy required, for fixed $|E(\Omega_o)|^2$ radiated, by the efficiency $\eta < 1$ of the antenna. *Assume* η constant throughout the band. The new values of G_o, B, and W are: $G_o' = \eta G_o$, $B' = B/\eta$, $W' = W/\eta$. To maintain W = 1 J we, therefore, have $G_o' = \eta^2 G_o$, $B' = B/\eta$ at any frequency. The B'-G_o' curve on Figure 2 would shift to the *left* of the dashed curve, hence loss—in first approximation—would decrease β_1 of (2), and *lower* the β_1-curve of Figure 3.

5. Maximum temporal $|r\mathcal{E}(\Omega_o,\tau)|^2$, $\tau = t - r/c$, for W = 1 J.

Comparison of the ω-domain expressions for $|rE(\Omega_o,\tau)|^2$ and $W_T(\Omega_o)$ shows that $|r\mathcal{E}|^2$ is maximized for $\tau = 0$ and

$$|r\mathcal{E}(\Omega_o,0)|^2 = \frac{1}{2}\sqrt{\frac{\mu_o}{\varepsilon_o}}\frac{G_o}{4\pi}\lim_{T\to 0}\frac{\lambda_n(B\omega T/2)}{T}, \qquad (8)$$

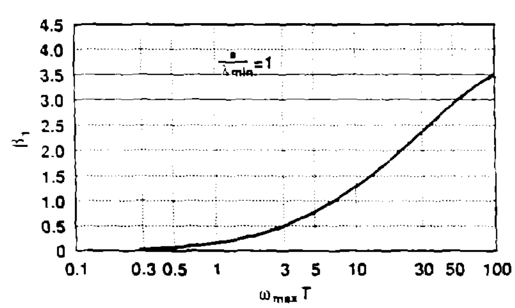

Fig. 3. β_1 = max $W_T(\Omega_o)/W$ vs. $\omega_{max}T$ from the dashed curve of Fig. 2.

λ_n being the n^{th} prolate spheroidal eigenvalue. For any set of values G_o, B, ω on the dashed curve of Figure 2 (8) is maximized for n = 1 and the result by Figure 1 is

$$|r\mathcal{E}(\Omega_o,0)|^2 = 4.77\ \omega G_o B = 4.77\ \omega_{max} \frac{G_o B}{1 + B/2} \qquad (9)$$

This last factor is maximized for $B(\omega) = 0.54$, $G_o(\omega) = 15.75$ on Figure 2, and so

$$|r\mathcal{E}(\Omega_o,0)|_{max} = 5.65 \sqrt{\omega_{max}} \qquad (a/\lambda_{min} = 1) \qquad (10)$$

Not surprisingly B = 0.54 is also the value which maximizes β_1 for $\omega_{max}T < 2.5$ in Figure 2. Loss degrades (9) also.

Comparison with [3, Figure 1] shows, for $\omega = 6\pi \times 10^8$ ($= \omega_{max}/(1 + B/2)$) considered there, (10) yields about half the values in that reference. This is partly because our $G_o = 15.75$ value is considerably less than the supergain limits of ka(ka + 2) assumed in the reference. Comparison with [1] for arrays indicates that (10) can be increased by driving the elements independently rather than with an (idealized) series circuit.

6. Conclusions

The formulas herein for maximum energy density $W_T(\Omega_o)$ per steradian radiated during (-T,T) and the maximum instantaneous $|r\mathcal{E}(\Omega_o, \tau = 0)|$ appear to be realistic for the constraints, with a spherical working volume of radius $a/\lambda_{min} = 1$, where $\lambda_{min} = 2\pi c/\omega_{max}$ and ω_{max} is the maximum signal frequency available. For larger a/λ_{min} one should construct best B-G_o curves, requiring examination of many possible antenna types within the volume. The forgoing procedure for studying directed radiation can be used, perhaps with modified scaling rules, with any B-G_o curve for a given a/λ.

References

[1] Y. W. Kang and D. M. Pozar, "Optimization of Pulse Radiation from Dipole Arrays for Maximum Energy in a Specified Time Interval," IEEE Trans. Ant. Prop., AP-34, Dec. 1986, pp. 1383-1389.

[2] J. Sahalos, "On the Optimimum Directivity of Antenna Consisting of Arbitrarily Oriented Dipoles," IEEE Trans. Ant. Prop., AP-24, May 1976, pp. 322-327.

[3] D. M. Pozar, D. H. Schaubert, and R. E. McIntosh, "The Optimum Transient Radiation for an Arbitrary Antenna," IEEE Trans. Ant. Prop., AP-32, June 1984, pp. 633-640. This article treats the case of a signal bandwidth much smaller than the antenna bandwidth, with $G(\Omega_o)$ replaced by ka(ka+2), the supergain limit of an antenna within a spherical working volume of radius a.

[4] R. M. Bevensee, "Fundamental Gain-Bandwidth-Volume Limitations of Antennas," Proc. ISAP 1985, pp. 389-392.

[5] R. F. Harrington, "Effect of Antenna Size on Gain, Bandwidth and Efficiency," Jour. Res. of the Nat. Bureau Stand.-D, 64D, Jan.-Feb. 1960, pp. 1-12.

[6] R. M. Bevensee, "Fundamental Limitations of Antennas in Terms of Gain, Bandwidth and Volume," Report to the Army, Ft. Monmonth, NJ, Feb. 1986 (unpublished).